ANALYSIS OF 3-D CYLINDRICAL STRUCTURES USING THE FINITE DIFFERENCE TIME DOMAIN METHOD

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Abstract

Recently, there has been a growing interest in using cylindrical transmission line structures in microwave applications. In this paper, the Finite Difference Time Domain (FDTD) method is used to characterize 3-dimensional cylindrical coplanar waveguide (CCPW) geometries. Specifically, a CCPW series stub and a three-section CCPW filter are studied theoretically and experimentally. Absorbing boundary conditions are employed to truncate the mesh at the end walls and the outer radial boundary.

1 INTRODUCTION

Cylindrical multiconductor transmission lines are of interest for many applications, in particular for new types of antennas and their feed lines. However, the design of passive components on cylindrical substrates is not an easy task, especially when there is no angular symmetry. Until now, 2-D and 3-D Integral Equation (IE) approaches, solved by the method of moments, have been the most commonly used techniques in this area [1]-[3]. Recently, a more versatile 2D-FDTD algorithm has been developed to analyze general cylindrical transmission lines, e.g., cylindrical microstrip line and cylindrical CPW (CCPW) [4].

The objective of this paper is to use the 3D-FDTD technique to analyze arbitrary cylindrical geometries, ranging from simple microstrip discontinuities to CPW filters. Our analysis can accurately model circuits in an open environment due to the use of an absorbing boundary condition to truncate the FDTD mesh at its outer radial boundary. Moreover, the problem of the singularity of the fields along the z-axis, which results if the ordinary FDTD equations are directly used to update all field components, is handled by discretizing the integral form of one of Maxwell's equations.

Two CCPW examples are included here and the numerical results are compared to measurements. The steps taken to build and measure the cylindrical structures under study are also described.

2 THEORETICAL FORMULATION

Starting from Maxwell's equations, in a source-free, lossless region, one can obtain 6 equations which relate the components of the electric field intensity $\bar{E}$ and magnetic field intensity $\bar{H}$ in cylindrical coordinates. For example,

$$-\mu_0 \frac{\partial H_r}{\partial t} = \frac{1}{r}\frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z}$$

Discretizing the 6 equations according to the 3D cell shown in Figure 1, one obtains 6 finite difference equations in the time domain. For example, the following difference equation is obtained from the above differential equation:

$$H_r^{n+\frac{1}{2}}(i,j,k) = H_r^{n-\frac{1}{2}}(i,j,k) + \frac{\triangle t}{\mu_0}\left[\frac{E_\phi^n(i,j,k+1) - E_\phi^n(i,j,k)}{\triangle z}\right]$$

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$$+ \quad \frac{\triangle t}{\mu_0} \left[\frac{E_z^n(i,j-1,k) - E_z^n(i,j,k)}{i \triangle\phi \triangle r} \right]$$

For numerical stability of the difference equations, the following condition should be imposed on the time step $\triangle t$:

$$c \triangle t \leq \frac{1}{\sqrt{(\frac{1}{\triangle r})^2 + (\frac{2}{\triangle r \triangle\phi})^2 + (\frac{1}{\triangle z})^2}}$$

where c is the velocity of light in free space.

It can be seen from Figure 1 that the field components E_ϕ, E_z and H_r must be evaluated along the z-axis (at $r = 0$). If the ordinary difference equations, similar to the one above, are directly used to compute $H_r(r = 0)$ and $E_z(r = 0)$, then singularities will exist [4]. Such singularities must be removed since the fields there should be finite in both the time and frequency domains. It can be shown that only E_z at $r = 0$ is needed to update all the other field components [4].

To determine E_z, the following integral form of Maxwell's equation in the time domain is used:

$$\oint_C \bar{H} \cdot d\bar{\ell} = \epsilon \int_S \frac{\partial \bar{E}}{\partial t} \cdot d\bar{s}$$

where C is a closed contour around the z-axis and S is the surface bounded by this contour. Using the closest closed path around the z-axis, i.e., the one at a distance of $(\triangle r/2)$, as our contour C, the following FDTD equation for E_z at $r = 0$ can be obtained:

$$\begin{aligned} E_z^{n+1}(0,j,k) &= E_z^n(0,j,k) \\ &+ \frac{4\triangle t}{\epsilon N_\phi \triangle r} \Sigma_{p=1}^{N_\phi} H_\phi^{n+\frac{1}{2}}(1,p,k) \end{aligned}$$

where ϵ is the permittivity of the medium just around the z-axis. Once E_z along the z-axis is known, the rest of the field components can be computed using the normal FDTD equations.

To be able to analyze open structures and account for radiation, an Absorbing Boundary Condition (ABC), similar to the one in [5], is used to truncate the mesh at its outer radial boundary. Moreover, the super-absorbing first-order Mur boundary condition is used at the front and back walls of the mesh in order to absorb the input Gaussian pulse.

3 RESULTS AND DISCUSSION

In order to test the developed cylindrical 3D-FDTD algorithm, a variety of cylindrical microstrip structures have been investigated and the results were compared with those obtained using other numerical methods. Specifically, the cylindrical microstrip open-end and gap discontinuities studied in [2, 3] using Integral Equation (IE) technique were analyzed using FDTD. The results obtained were in very good agreement with the published IE results.

One of the main objectives of this research is to study the newly proposed cylindrical coplanar waveguide (CCPW) [6] shown in Figure 2. The line parameters, ϵ_{eff} and Z_0, have been analyzed in [4] using the 2D-FDTD technique.

In order to validate the 3D-FDTD code, several cylindrical CPW lines and discontinuities have been fabricated and measured. The realization of geometries with a small radius of curvature was achieved by patterning the circuits on a 3 *mil*- thick copper-clad Teflon substrate using photolithographic techniques. The substrate is then mounted to a flat support, and $FeCl$ etchant is used to generate the patterns. After etching, the thin substrate is transferred to a Teflon rod or tube of the desired dimensions. SMA-type connectors are then mounted to the ends of the rod, and connected to the transmission line by soldering.

Two of the 3-D structures that have been characterized are a series short-end stub printed in the ground plane and a three-section band-pass filter, illustrated in Figures 3 and 4, respectively. The theoretical and experimental results for the series stub are given in Figure 5. Except for the discrepancies around 9 GHz, which are due to calibration error, the agreement is very good. Similar agreement is obtained for the filter, as shown in Figure 6. The FDTD parameters used to model both structures are as follows: $\triangle r$=0.3048 mm, $\triangle z$=0.4 mm, N_r=41, N_ϕ=100, and $\triangle t$=3.1256E-14 sec.

4 SUMMARY

A three dimensional Finite Difference Time Domain (3D-FDTD) algorithm in cylindrical coordinates has been developed. The algorithm has been validated by comparing the results for several representative cylindrical structures to published numerical data. Experimental validation has also been presented herein.

5 ACKNOWLEDGEMENTS

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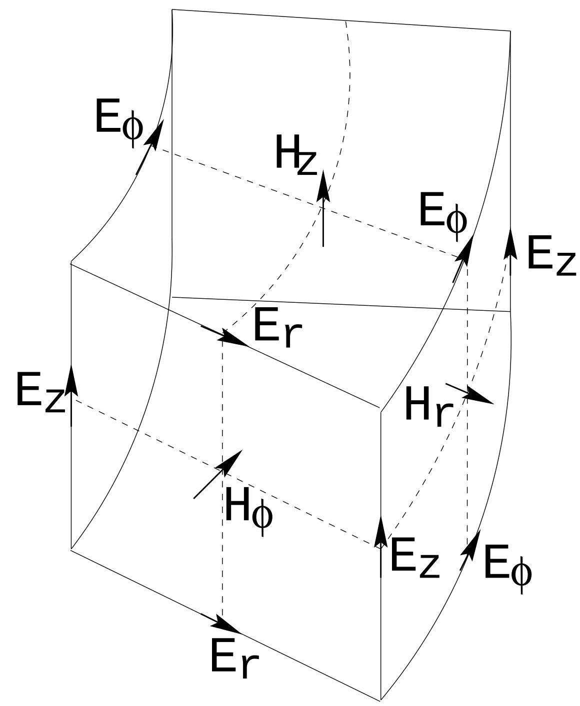

Figure 1: A typical 3D-FDTD cell in the cylindrical coordinate system.

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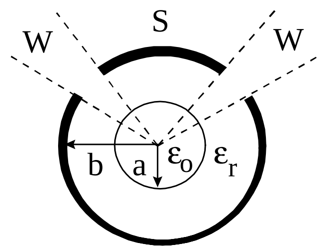

Figure 2: Cross-section of a cylindrical CPW line.

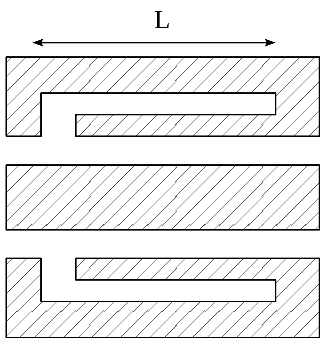

Figure 3: A series short-end stub printed in the ground plane. The center conductor width is 2.0 mm, all slots and gaps are 0.4 mm, and the stub length (L) is 14.0 mm.

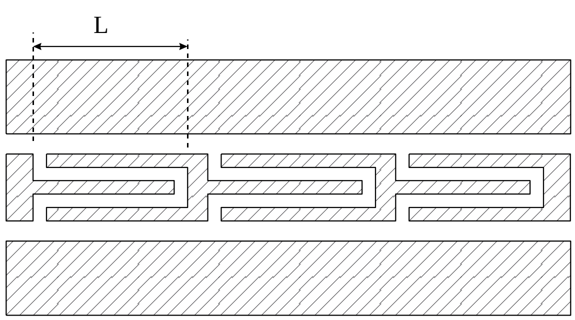

Figure 4: A three-section bandpass filter. The center conductor width is 2.0 mm, all slots and gaps are 0.4 mm, and the stub length (L) is 14.0 mm.

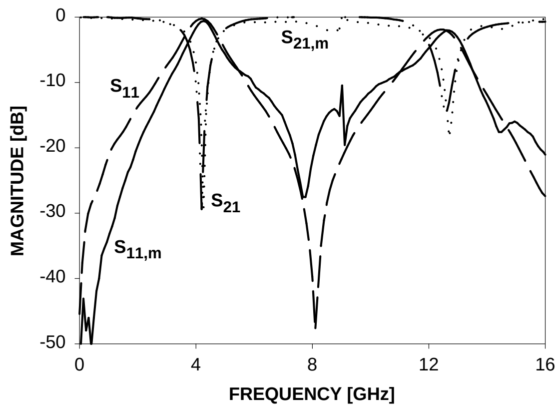

Figure 5: Measured ($S_{11,m}$, $S_{21,m}$) and FDTD S-parameters (S_{11}, S_{21}) of the cylindrical CPW, series short-end stub (a=0 mm, b=6.4 mm, S=2.0 mm, W=0.4 mm, and ϵ_r=2.01).

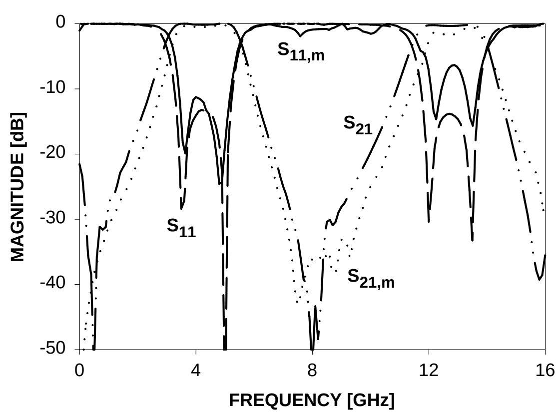

Figure 6: Measured ($S_{11,m}$, $S_{21,m}$) and FDTD S-parameters (S_{11}, S_{21}) of the three-section cylindrical CPW bandpass filter.